Probability for Finance and Insurance 2018/2019

1. Dates: : 30/05 in the afternoon, 07/06, 12/06, 19/06 and 27/06. in the morning. Some slots in September, too, but not very many. The exam is oral and takes place in my office. You sign up to a particular date by sending me an e-mail (maslow@karlin.mff.cuni.cz). **I have to confirm its receipt and specify the time (approximately).**   The number of students for each exam is limited, so don't hesitate. If you don't plan to come this semester, I would appreciate if you let me know.

Requirements:

Basic Notions and Theorems (numbering is the same as at the lecture):

1. Basics from the theory of random processes : Conditional expectations w.r.t. sigma-algebra, its basic properties and interpretation., Daniell-Kolmogorov (T 2.1), the principle of construction of a random process by its finite-dimensional distributions. Kolmogorov-Chentsov continuity test (T 2.4) and application to the existence proof of the Wiener process (the basic idea).

2. Martingales: Stochastic basis, filtrations, usual conditions. Adapted and progressively measurable processes and relations between them. The concept of (sub-, super-) martingales, L^2- martingales, local martingales, examples. Basic properties of martingales. Stopping times and their basic properties, examples (T 4.9). Measurability of the stopped process (T 4.11). Optional Sampling Thm. (T 4.12). Maximal inequalities (T. 4,13). Concept of increasing natural (predictable) process, the class (DL). Doob-Meyer decomposition of a right-contuinuous submartingale (T 4.18) and applications in the definition of quadratic variation (P 4.19). Examples of D-M decompositions of martingales. Quadratic variation as a limit of partial sums (T 4.21)

3. Stochastic Analysis: Wiener process - motivation, definition and basic properties, what is the relation the the concept of white noise? Stochastic integral - definition and a basic properties (P 5.3, 5.8), connection to the Lebesgue - Stieltjes integral. Stochastic differential and Ito formula - formulation and applications in the examples. Stochastic bilinear equation (dX= f(t)X dt + g(t)X dW) and geometric Brownian motion, the formula for solution . Linear equation (dX=f(t)X dt + g(t)dW), the formula for solution (variation of constants) .

Proofs: Show that Wiener and the "compensated" Poisson process are martingales and find their Doob-Meyer decomposition, proof of Ito isometry for step functions (P 5.3). Intuitive derivation of Ito formula. Proof of the formulas for solutions to bilinear and linear equations (the two examples following the Ito lemma).

Obviously, the knowledge of all definitions and basically all statements presented during the lecture is necessary for the exam to be successful. The level of understanding of the topic is important as well.